Size does not matter. . . sometimes

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Abstract

Cosmological fine-tuning has traditionally been associated with the narrowness of the intervals in which the parameters of the physical models must be located to make life possible. A more thorough approach focuses on the probability of the interval, not on its size. We present a framework to measure tuning that, among others, deals with the normalization problem, assuming that the prior distribution belongs to a class of maximum entropy (maxent) distributions. By analyzing an upper bound of the tuning probability for this class of distributions the method solves the so-called weak anthropic principle, and offer a solution, at least in this context, to the well-known lack of invariance of maxent distributions. The implication of this approach is that, since all mathematical models need parameters, tuning is not only a question of natural science, but also a problem of mathematical modeling. Therefore, whenever a mathematical model is used to describe nature, not only in physics but in all of science, tuning is present. And the question of whether the tuning is fine or coarse for a given parameter — if the interval in which the parameter is located has low or high probability, respectively — depends crucially not only on the interval but also on the assumed class of prior distributions. Novel upper bounds for tuning probabilities are presented [Dıéaz-Pachón et al. 2022].

Problems measuring fine-tuning

- **Normalization:** Bernoulli's Principle of Insufficient Reason cannot be invoked since relevant fine-tuning spaces often have infinite cardinality [McGrew et al. 2001].
- **Weak anthropic principle:** We live in a habitable universe, therefore we are constrained in our sample of size 1 to only observe values that permit life [Bostrom 2002].
- **A single maxent distribution** is considered [McGrew 2018].
- **Lack of invariance of maxent:** Maximum entropy distributions are not invariant to transformations [Koperski 2005].

Four-step procedure [Díaz-Pachón et al. 2021]

1. **Determine the sample space** Ω.
2. **Determine the moments constraints of the distribution** $E[M_i(X)] = \theta_i$ **for** $i = 1, \dots, d$
3. **Find the family** $\mathcal{F}$ **of maxent distributions** F.
4. **Find the maximum probability.** Take $\text{TP}_{\text{max}} = \max\{F(I) : F \in \mathcal{F}\}$.

Theorem 1: Maximal tuning probabilities given certain constraints

Ω	$\mathcal{F}$	Θ	Constraint	TP_{max}
$\mathbb{R}^+$	Scale	$\mathbb{R}^+$	$\epsilon \ll 1$	$2\epsilon C_1$
	Form and scale	$\mathbb{R}^+ \times \mathbb{R}^+$	None	1
	Form and scale	$\mathbb{R}^+ \times \mathbb{R}^+$	SNR $\leq S$, $\epsilon \ll 1$, $\epsilon\sqrt{S} \ll 1$, $S \gg 1$	$2\epsilon\sqrt{S/(2\pi)}$
$\mathbb{R}$	Scale	$\mathbb{R}^+$	$0 \notin I_X$, $\epsilon \ll 1$	$2\epsilon C_1$
	Scale	$\mathbb{R}^+$	$0 \in I_X$	1
	Location	$\mathbb{R}$	$C_3 \ll 1/\epsilon$, $\epsilon \ll 1$	$2\epsilon C_3$
	Location	$\mathbb{R}$	None	1
	Location and scale	$\mathbb{R} \times \mathbb{R}^+$	SNR $\leq S$, $\epsilon \ll 1$, $\epsilon\sqrt{S} \ll 1$	$2\epsilon(C_3\sqrt{S} + C_1)$
	Location and scale	$\mathbb{R} \times \mathbb{R}^+$	None	1

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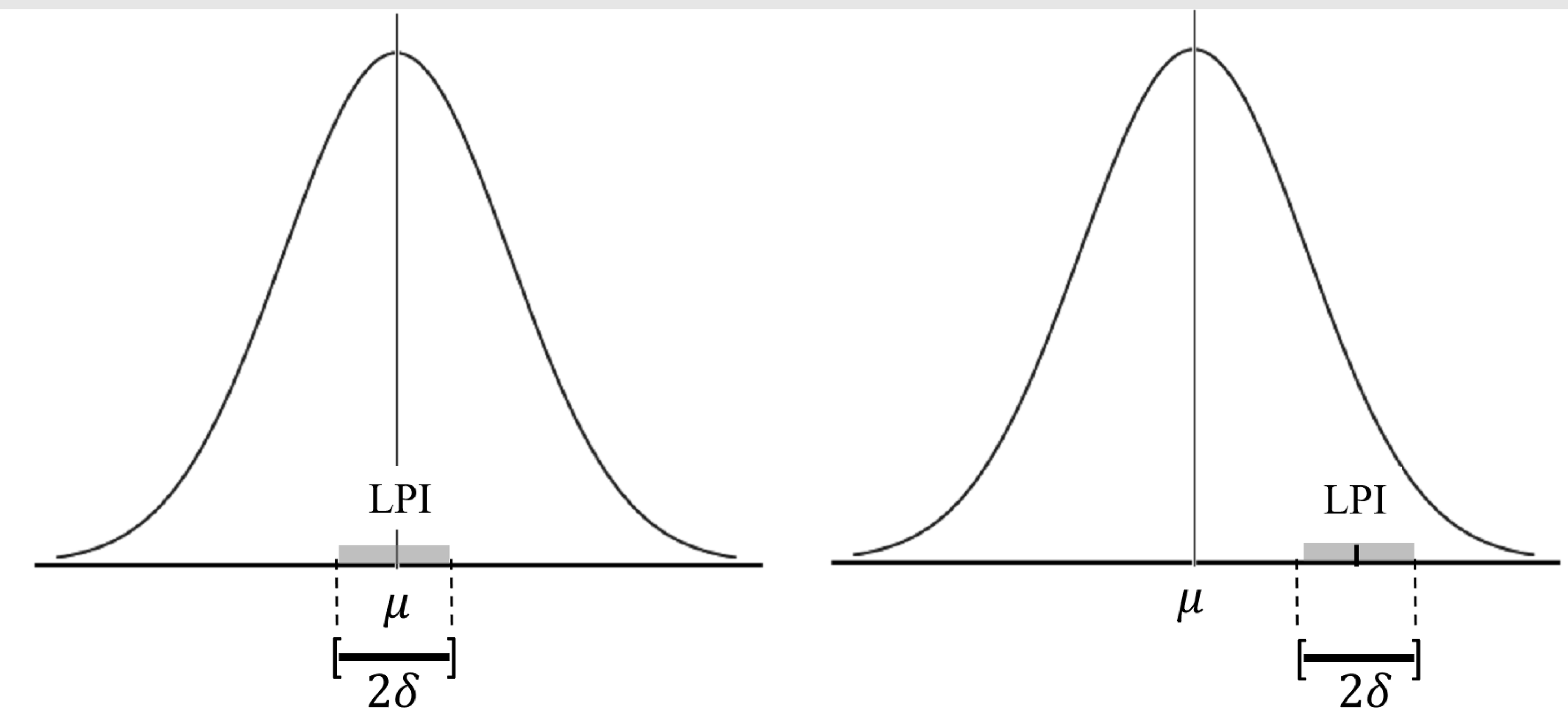

Figure 1. **Example with normal distribution:** When the variance σ^2 of the prior distribution of X approaches 0, the normal distribution approaches a Dirac delta measure at μ; thus $\mu \in I$, implies $\text{TP}_{\text{max}} = 1$ (left). On the other hand, when $\mu \notin I$, TP_{max} will go to zero either when $\sigma \to 0$ or when $\sigma \to \infty$. Therefore TP_{max} is strictly less than 1 (right). For the figure at the left, the **detected** tuning is coarse, whereas for the figure at the right it is fine.

- Left figure: very small intervals with very large probability.
- Tails of distribution: large intervals with small probability.
- Probability matters, not size!

Implications of the procedure

- **Normalization solved.** The method does not assume the principle of insufficient reason, but the more general principle of maximum entropy.
- **Weak anthropic principle solved.** A family of maxent is considered, not only that of our universe.
- **Lack of invariance of maxent distributions solved**. By an appropriate selection of constraints in Step 2.
- **Unique maxent distribution solved.** A family $\mathcal{F}$ of maxent distributions is considered in Step 3.
- **No false positives.** When the method detects fine-tuning (TP_{max} is small), there is fine-tuning.
- **False negatives.** When coarse-tuning is detected (TP_{max} is large), the method is inconclusive.

Example 1: Critical density of the universe

According to Paul Davies 1982, the critical density of the universe ρ_{crit} cannot take values outside the interval

$$I_{\rho_{\text{crit}}} = \rho_{\text{crit}}[1 - 10^{-60}, 1 + 10^{-60}].$$

- Since the density cannot be negative, $\Omega = \mathbb{R}^+$.
- For $\epsilon = 10^{-60}$, $\epsilon \ll 1$.

1. **Scale family:** There is fine-tuning, since $\text{TP}_{\text{max}} = 2 \times 10^{-60} C_1$.
2. **Form and scale (I)**. Provided the signal-to-noise ratio of the prior is bounded above by S and $\sqrt{S} \ll 10^{60}$, then $\text{TP}_{\text{max}} = 2 \times 10^{-60}\sqrt{S/2\pi}$, there is fine-tuning.
3. **Form and scale (II)**. There is coarse-tuning detected, since $\text{TP}_{\text{max}} = 1$.

Example 2: Gravitational force

According to Davies 1982, when observing the ratio X of the gravitational constant G_{obs} to the contribution from vacuum energy to the cosmological constant Λ_{vac}, gravitation cannot fall outside the life-permitting interval

$$I_X = x_{\text{obs}}\left[1 - 10^{-100}, 1 + 10^{-100}\right].$$

Then taking $\epsilon = 10^{-100}$,

- **If gravitation can only be attractive**, $\Omega = \mathbb{R}^+$.
- **If gravitation can also be repulsive** [Barnes 2012], $\Omega = \mathbb{R}$, and
  1. **For the scale family**, since $0 \notin I_X$, there is fine-tuning: $\text{TP}_{\text{max}} = 2 \times 10^{-100} C_1$
  2. **For the location family** there is fine-tuning when $C_3 \ll 10^{100}$, since $\text{TP}_{\text{max}} = 2 \times 10^{-100} C_3$.
  3. **For the location and scale family** there is fine-tuning when $S \ll 10^{200}$, since $\text{TP}_{\text{max}} = 2 \times 10^{-100}(C_3\sqrt{S} + C_1)$.
- In all other cases coarse-tuning is detected.

Example 3: Amplitude of primordial fluctuations

According to Martin Rees 2000, the amplitude of the primordial fluctuations must be in the interval

$$I_Q = \left[10^{-6}, 10^{-5}\right].$$

Since the amplitude cannot be negative, $\Omega = \mathbb{R}^+$. Also $\epsilon \gg 0$.
The theorem does not apply!
However, assuming an exponential distribution,

$$\text{TP}_{\text{max}} \approx 0.697. \quad (1)$$

Conclusions

1. **Versatile approach.** It works with the current constants of nature and standard models . . . or with others.
2. **A problem of mathematical modeling.** Tuning analysis is a problem of mathematical modeling, not exclusively pertaining to cosmology.
3. **Reconfiguration of the tuning problem.** Tuning analysis is about probability, not the size of intervals.

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